



Interpretation of Small Punch Creep Tests Data for Ductile Materials

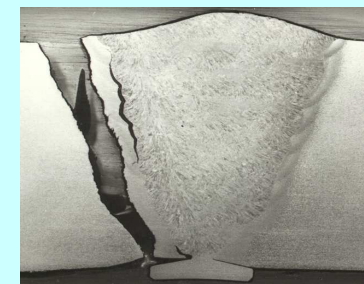
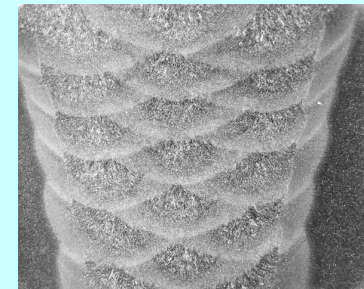
T. H. Hyde and W. Sun

**Dept. of Mech., Mats. & Manuf. Engineering
University of Nottingham
Nottingham UK**



Introduction

- ❑ The small punch (SP) creep test method has been used in determining creep properties for welds, and the remaining creep life of, for example, steam pipelines, boilers and headers.
- ❑ The responses of a SP creep tests are related to the punch/specimen dimensions and the elastic-plastic and creep behaviour of the test material, under contact and large deformation conditions, at elevated temperature.
- ❑ The main challenge is to infer the uniaxial creep properties from the experimental SP creep test data.
- ❑ This presentation starts with a description of the reference stress method and the main restrictions for it to be directly applied to the SP test method.
- ❑ Based on approximate analyses and observed, typical experimental data, reasons for the apparent correlation between the minimum creep deformation rate from a SP test and the minimum creep strain rate from a uniaxial creep test are provided and a systematic approach for interpreting data from a SP test is addressed.





Basis of Reference Stress Method

□ The reference stress method allows the creep deformation of a component at a particular load to be related to the strains obtained from a single uniaxial creep test at a “reference stress”.

□ For some components it is possible to obtain analytical solutions which relate the displacement rate to the load, material properties and geometry, e.g., for a component made from a Norton material,

$$\dot{\Delta} = f(P, B, n, \text{dim}) \quad \dot{\epsilon}_c = B\sigma^n$$

□ Inspection of the solutions for components, for which analytical solutions can be obtained, e.g. the cantilever beam (Appendix 1), shows that they are of the form:

$$\dot{\Delta} = f_1(n)f_2(\text{dim})B(\sigma_{nom})^n$$

□ The basis of the reference stress method is that a constant, α , can be chosen such that $f_1(n)/\alpha^n$ is practically independent of n ; this particular value of α is designated η . Hence, the above equation can be written as:

$$\dot{\Delta} = DB(\eta\sigma_{nom})^n = D\dot{\epsilon}(\sigma_{ref})$$

where $\eta\sigma_{nom}$ is the so-called reference stress, σ_{ref} . The quantity D (the reference multiplier) is a constant ($= f_1(n).f_2(dim)/\eta^n$), which has a unit of length if Δ is a displacement.



Basis of Reference Stress Method...

□ The requirement is to obtain the value of α which renders D to be practically independent of n (when $\eta = \alpha$). Various methods may be used to determine the η -values.

□ If an analytical solution does not exist for the particular component of interest, a series of FE solutions with different n -values can be used to determine the reference stress, σ_{ref} , and reference multiplier, D .

□ Alternatively, approximate reference stresses and multipliers can be obtained from limit load and linear elastic solutions for the component:

$$\sigma_{ref} \approx \frac{P}{P_L} \sigma_y \quad D \approx \frac{\Delta^e}{(\sigma_{ref} / E)}$$

□ Therefore, if the reference parameters (σ_{ref} and D) are known, and the deformation can be measured, the equivalent uniaxial creep strain rate at σ_{ref} for the component and load can be determined:

$$\dot{\epsilon}(\sigma_{ref}) = \frac{\dot{\Delta}}{D}$$

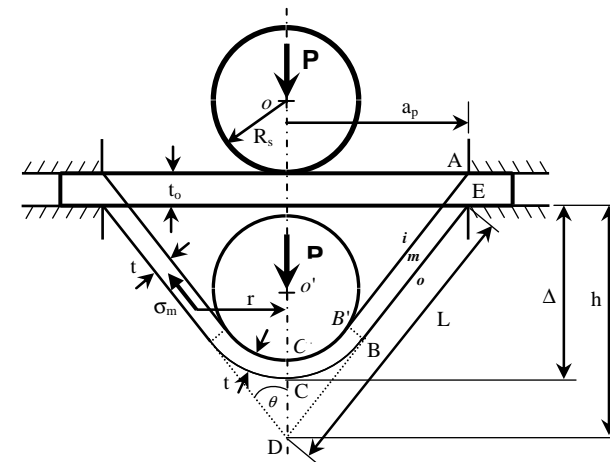
where D is in effect an “equivalent gauge length”.



Typical SP Creep Test Behaviour

The SP test is highly complex and involves the interactions between a number of nonlinear processes.

- (a) **Contact:** the contact area between the specimen and the punch increases as the "constant load" creep test progresses (friction may be important).
- (b) **Non-linear material behaviour:** in general the elastic-plastic and the creep strains are non-linearly related to the stress state.
- (c) **Large deformation:** the specimen starts as a flat plate and ends up being approximately conical in shape with a part-spherical shaped end.
- (d) **Large strains:** for most materials tested using the SP creep test method, the corresponding failure strains from uniaxial tests are in excess of ~ 25%. There is often evidence of localised "necking" at or near the edge of contact between the specimen and the punch, at which position the strains are significantly greater than the general strain level in the specimen as a whole.



Initial and deformed (assumed constant thickness) shape of the SP test specimen. Three important dimensions:

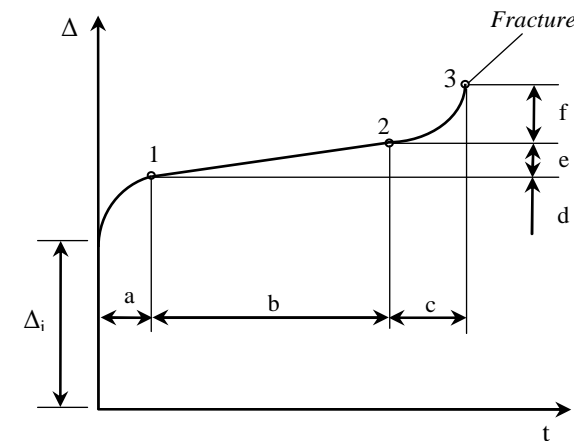
R_s , a_p and t_0 .



Typical SP Creep Test Behaviour...

The highly non-linear deformation behaviours experienced during a SP creep test cause the interpretation of the results to be difficult.

- a) Reducing deformation rate;
- b) Approximately constant deformation rate;
- c) Increasing deformation rate;
- d) Deformation occurring during reducing deformation rate;
- e) Deformation occurring during "constant" deformation rate; and
- f) Deformation occurring during increasing deformation rate.



Schematic representation of displacement versus time curve from SP creep test, showing different deformation regions (where Δ_i is the instantaneous elastic and plastic deformation).



Approximate Theoretical Model

Several approximate, "empirical" relationships exist which relate the SP deformation to a "mean" uniaxial strain and SP load to a "mean" uniaxial stress.

□ Li and co-workers have used the Chakrabarty model to derive equations relating the equivalent strain, ε_{eq} , at the edge of contact between the specimen and the punch to the overall displacement, Δ , and the membrane stress, σ_m , for an applied force, P , to the displacement. For a specimen with $a_p = 2\text{mm}$, $R_s = 1.25\text{mm}$ and $t_0 = 0.5\text{mm}$, the relationships are:

$$\varepsilon_{eq} = 0.17959\Delta + 0.09357\Delta^2 + 0.0044\Delta^3 \quad (7)$$

$$\frac{P}{\sigma_m} = 1.72476 \Delta - 0.05638 \Delta^2 - 0.17688 \Delta^3 \quad (8)$$

□ An approximate analysis, which allows the "general" strain levels to be estimated, has been obtained by Hyde and co-workers. The "general" strain level and the membrane stress are:

$$\bar{\varepsilon}_m = \ln \left(\frac{1}{\sin \theta} - \frac{R_s}{a_p \tan \theta} + \frac{R_s}{a_p} \left(\frac{\pi}{2} - \theta \right) \right) \quad (10)$$

$$\sigma_m = \frac{P}{2\pi R_s t_0} \frac{\sqrt{1 + \frac{1}{\tan^2 \theta}}}{\cos^2 \theta} \quad (12)$$

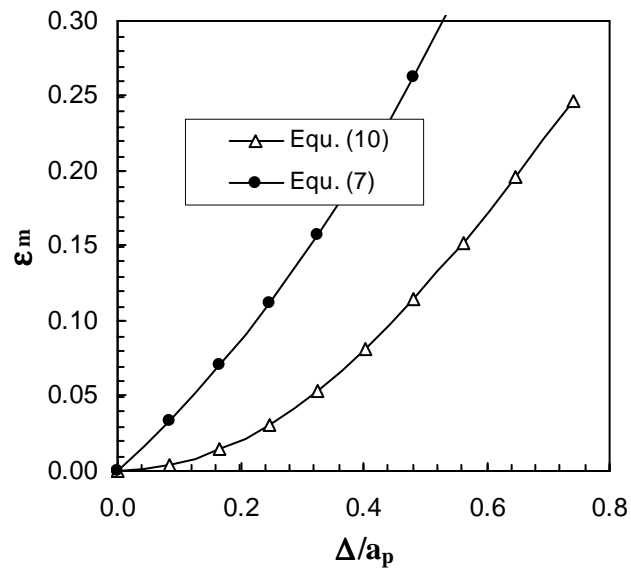
Chakrabarty J. (1970). A theory of stretch forming over hemispherical punch heads. *Int. J. Mech. Sci.* **12**, 315-325.

Li Y. Z. and Šturm R. (2008) Determination of creep properties from small punch test. *Proc. ASME Pressure Vessels and Piping Division Conference*. July 27-31, 2008, Chicago, Illinois, USA.

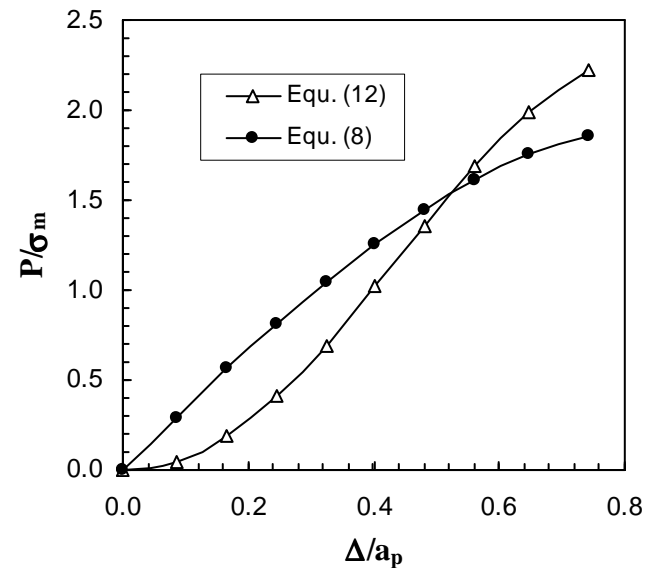
Hyde T. H., Miroslav S., Sun W. and Hyde C. J. (2010). On the interpretation of results from small punch creep test. *J. of Strain Analysis* **45** (3), 141-164.



Approximate Theoretical Model...



Variation of ϵ_m with Δ/a_p for $a_p = 2\text{mm}$ and $R_s = 1.25\text{mm}$.



Variation of P/σ_m with Δ/a_p for $a_p = 2\text{mm}$ and $R_s = 1.25\text{mm}$.



Relationship between Min Deformation Rate and Secondary Creep Rate...

□ Experimental data indicates that when min creep strain rate versus σ from uniaxial creep data is compared with the min deformation rate versus P from SP creep data on log-log scales, the gradients are approximately the same for some materials, which are equal to the n -values.

□ An indication as to why the n value obtained from SP tests appears to be correct approximately can be obtained from a large deformation ductile analysis of a uniaxial specimen for which small strains are governed by the Norton law. The large strain analysis results in:

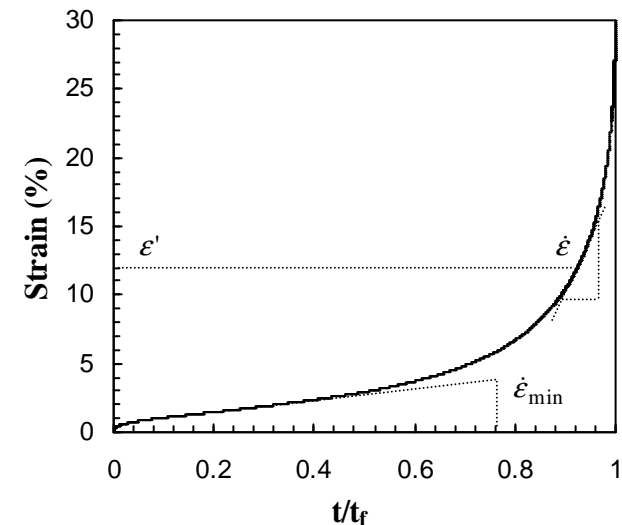
$$\dot{\epsilon} = (1 + \epsilon')^n B \sigma_o^n$$

□ Hence a plot of creep strain rate versus σ_o on log-log scales will have a gradient of n , where σ_o is the nominal stress based on the original dimensions;

$$\log(\dot{\epsilon}) = n \log(1 + \epsilon') + \log(B) + n \log(\sigma_o)$$

where the intercept of the log-log plot would be

$$[n \log(1 + \epsilon') + \log(B)]$$

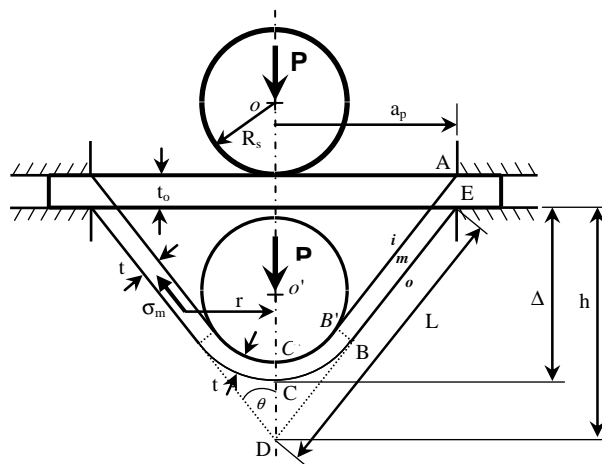


A typical uniaxial creep strain curve of a P91 steel tested at 650° C.

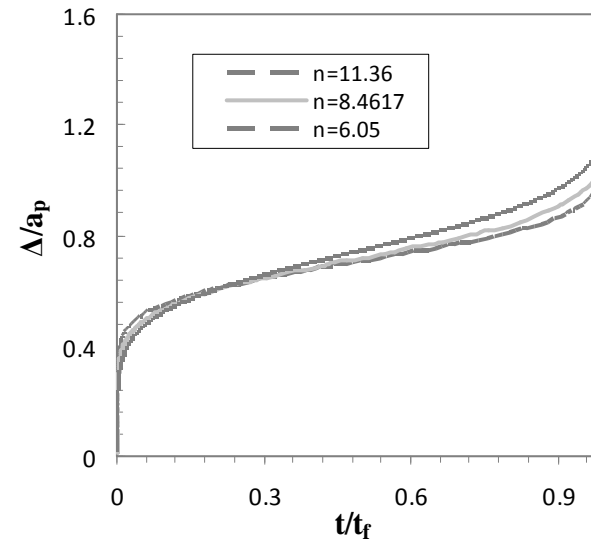


Finite Element Modelling

- A series of FE analyses have been performed in order to verify and assess the proposed method for using the deformation rate from SP test to obtain constants in Norton's law.
- Typical FE predictions were obtained from large deformation, large strain, contact analyses for materials using Norton's law with n-values of 6.05, 8.46 and 11.36.



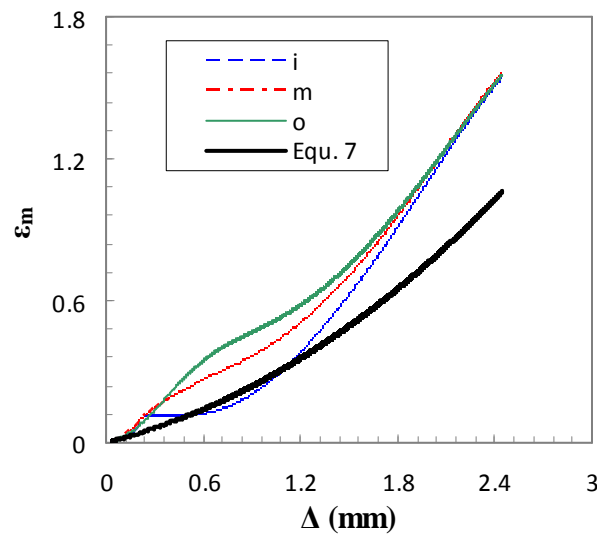
Parameter definitions of Initial and deformed shape of the SP test specimen.



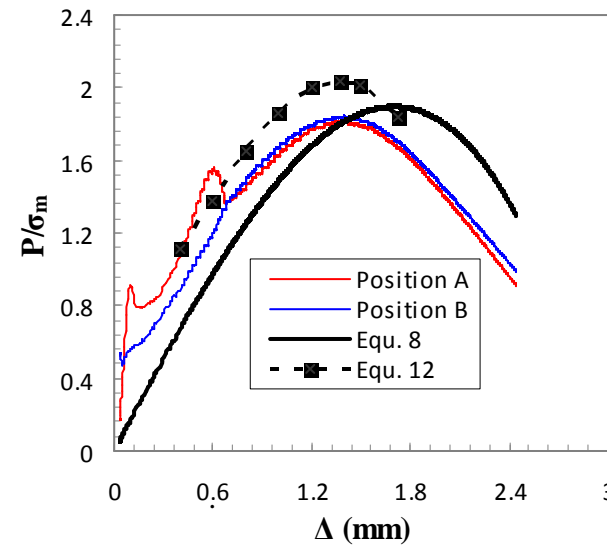
Δ versus t/t_f for Norton law creep behaviour with $n=6.05, 8.4617, 11.36$.



Finite Element Modelling...



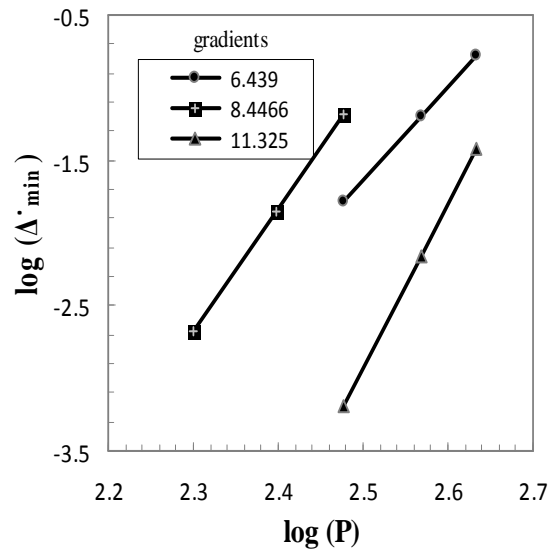
ϵ_m versus Δ at inside (i), middle (m) and outside (o) of the disc thickness at Position B; $n = 6.05$.



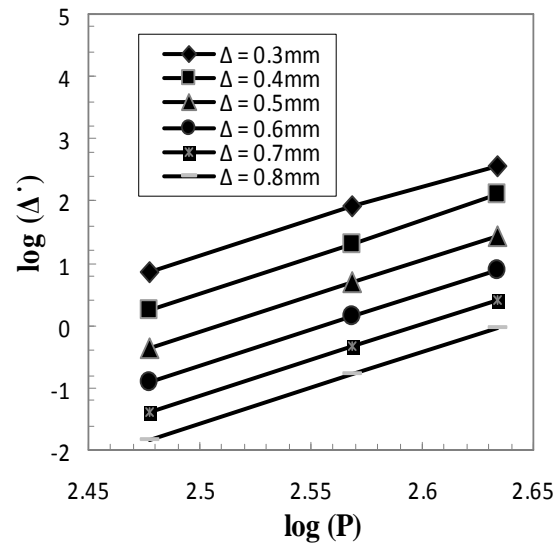
Variation of P/σ_m with Δ (middle, m); $n = 6.05$.



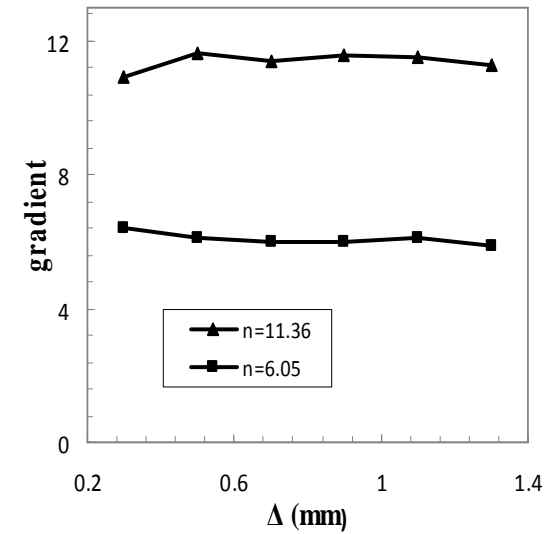
Finite Element Modelling...



Min deformation rate versus P (log-log scale) based on Norton's law ($n = 6.05$, $B = 1.88 \times 10^{-18}$; $n = 8.4617$, $B = 1.09 \times 10^{-20}$; $n = 11.36$, $B = 5.38 \times 10^{-29}$).



Min deformation rate versus P (log-log scale) for different displacements ($n = 11.36$, $B = 5.38 \times 10^{-29}$).



Variation of gradient of log min deformation rate versus log(P) with Δ.



Determination of Reference Parameters η and β

□ The minimum displacement rates for a particular material occurs at about the same value of Δ , irrespective of load level.

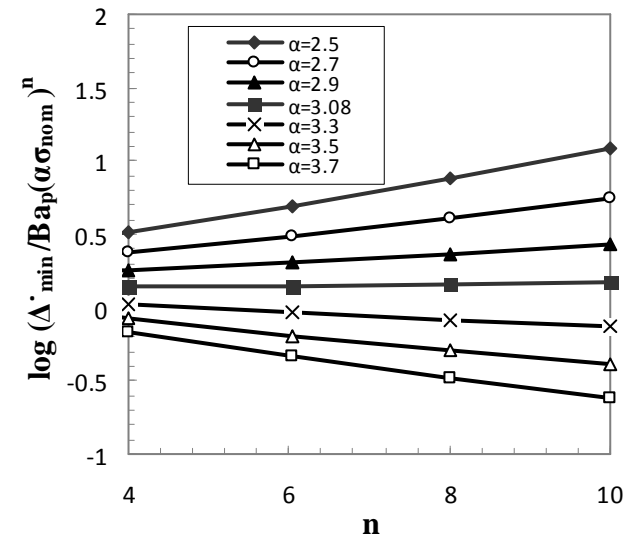
$$\dot{\Delta} = DB(\eta\sigma_{\text{nom}})^n = \beta a_p B(\eta\sigma_{\text{nom}})^n$$

FE solutions have been obtained for various n -values.

□ By plotting $\log \beta$ versus n for various values of α , the value of α which produces a horizontal line enables the α -value related to the reference stress, i.e. the η -value, to be identified, while the intercept on the vertical axis, which is $\log(\beta)$, allows the reference multiplier ($D = a_p\beta$) to be determined.

□ The σ_{nom} value can be arbitrarily chosen; in the present case, for convenience, it is taken to be:

$$\sigma_{\text{nom}} = \frac{P}{2\pi a_p t_0}$$



Variations of $\log\left[\frac{\dot{\Delta}_{\text{min}}}{a_p B (\alpha \sigma_{\text{nom}})^n}\right]$ with n for various α -values.



Reference Parameters η and β

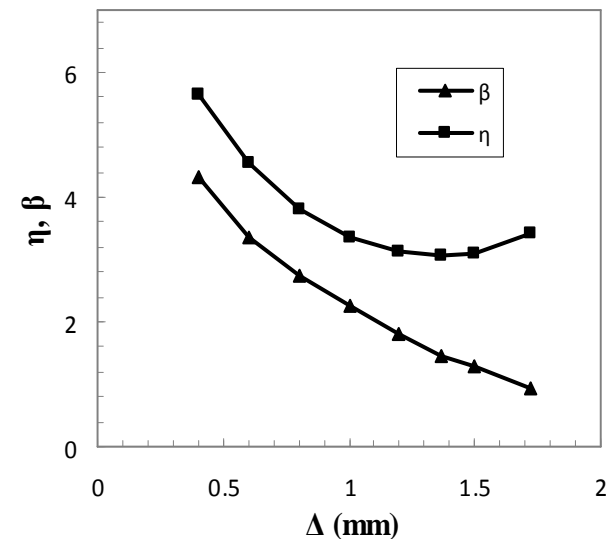
(for $R_s = 1.25\text{mm}$, $a_p = 2\text{mm}$ and $t_o = 0.5\text{mm}$)

□ Applying the same technique to obtain the η and β values for other values of Δ allows the variation of η and β with Δ to be obtained.

□ The effective gauge length, $D = \beta a_p$, reduces with increasing Δ . Also, there is a minimum value of η , i.e. $\eta_{\min} = 3.08$, which occurs when the deformation rate is a minimum.

□ Using $\sigma_{\text{ref}} = \eta \sigma_{\text{nom}}$, $\frac{P}{\sigma_{\text{ref}}} = \frac{2\pi a_p t_o}{\eta}$

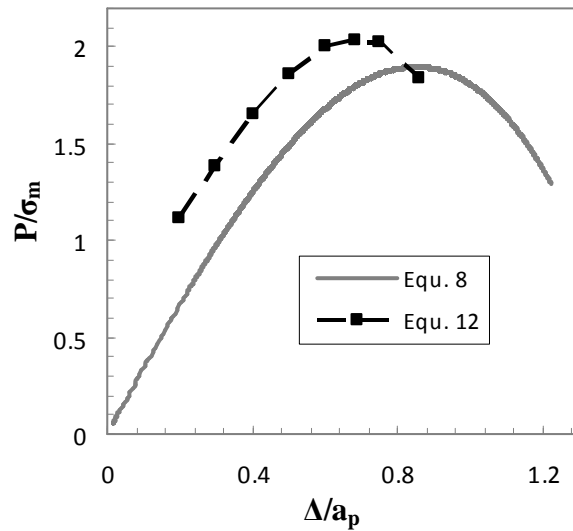
□ Then for $a_p = 2\text{mm}$, $R_s = 1.25\text{mm}$ and $t_o = 0.5\text{mm}$, the maximum value of P/σ_{ref} , which occurs at $\Delta/a_p \approx 0.7$, is 2.04 mm^2 . This is close to the maximum $P/\sigma_m = 1.89 \text{ mm}^2$ and $\Delta/a_p = 0.8$ predicted on the basis of the Chakrabarty membrane model.



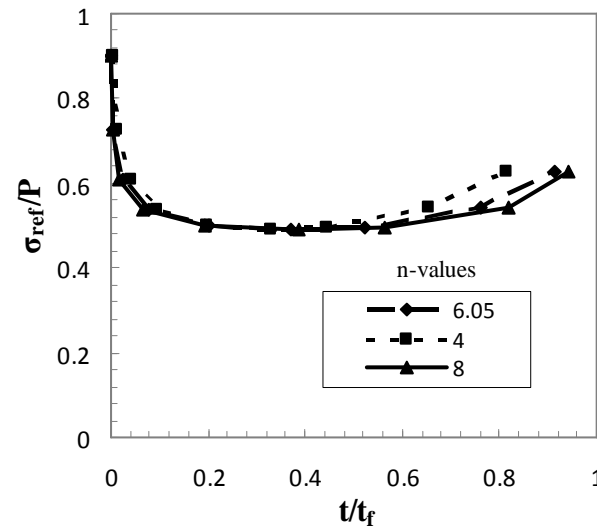
Variation of η and β with Δ for $a_p = 2\text{mm}$, $R_s = 1.25\text{mm}$ and $t_o = 0.5\text{mm}$.



Variation of P/σ_m with Δ/a_p and σ_{ref}/P with t/t_f



Variation of P/σ_m with Δ/a_p ($a_p = 2\text{mm}$, $R_s = 1.25\text{mm}$ and $t_0 = 0.5\text{mm}$).



Variation of σ_{ref}/P with t/t_f for different n-values.



Conclusions

□ Reference parameters, η and β , which relate the test conditions (load and specimen dimensions) and test results (deformation versus time) to corresponding uniaxial creep strain versus time data have been established for a typical geometry ($a_p = 2.0\text{mm}$, $R_s = 1.25\text{mm}$ and $t_o = 0.5\text{mm}$).

□ For the majority of a SP test duration, the reference stress is related to the applied load, P , via the relationship $\sigma_{ref} = 0.512P$, for creep ductile materials, where P has units of N and σ_{ref} has units of MPa.

□ The minimum displacement rate in a SP test relates to the strain rate at some position within the tertiary creep region, i.e. not directly to the minimum strain rate in a uniaxial creep test.

□ However, the strain rate related to the minimum displacement rate can be determined by using the equation with $D = \beta a_p$ using the known β -values, i.e.

$$\dot{\epsilon}(\sigma_{ref}) = \frac{\dot{\Delta}}{D}$$

□ The minimum creep strain rate obtained from a uniaxial creep test is approximately related to the creep strain rate at a strain of ϵ' in the tertiary region, for a ductile material, via the relationship:

$$\dot{\epsilon}(\epsilon = \epsilon') = \dot{\epsilon}_{min} (1 + \epsilon')^n$$



Acknowledgements

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